LA-UR-01-2371
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Title:

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Submitted to:

http://lib-www.lanl.gov/la-pubs/00796058.pdf

Structural Health Monitoring of Welded Connections

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ABSTRACT

Structural health monitoring is the implementation of a damage detection strategy for aerospace, civil and mechanical engineering infrastructure. Typical damage experienced by this infrastructure might be the development of fatigue cracks, degradation of structural connections, or bearing wear in rotating machinery. The goal of the research effort reported herein is to develop a robust and cost-effective monitoring system for welded beam-column connections in a moment resisting frame structure. The structural health monitoring solution for this application will integrate structural dynamics, wireless data acquisition, local actuation, micro-electromechanical systems (MEMS) technology, and statistical pattern recognition algorithms. This paper provides an example of the integrated approach to structural health monitoring being undertaken at Los Alamos National Laboratory and summarizes progress to date on various aspects of the technology development.

INTRODUCTION

The process of implementing a damage detection strategy for aerospace, civil and mechanical engineering infrastructure is referred to as *structural health monitoring (SHM)*. Here *damage* is defined as changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the system's performance. The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors, the extraction of damage-sensitive features from these measurements, and the statistical analysis of these features to determine the current state of system health. For long term SHM, the output of this process is periodically updated information regarding the ability of the structure to perform its intended function in light of the inevitable aging and degradation resulting from operational environments. After extreme events,

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such as earthquakes or blast loading, SHM is used for rapid condition screening and aims to provide, in near real time, reliable information regarding the integrity of the structure.

Current SHM methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiograph, eddy-current methods and thermal field methods (Doherty, 1987). All of these experimental techniques require that the vicinity of the damage is known *a priori* and that the portion of the structure being inspected is readily accessible. The need for quantitative *global* damage detection methods that can be applied to complex structures has led to research into SHM methods that examine changes in the vibration characteristics of the structure. The basic premise of these global SHM methods is that damage will alter the stiffness, mass or energy dissipation properties of a system, which, in turn, alter the measured dynamic response of the system. Summaries of this research can be found in recent review articles (Doebling, 1998; Housner, 1997). To date, most global SHM techniques proposed in these references examine changes in modal properties (resonant frequencies, mode shapes), or changes in quantities derived from modal properties. Drawbacks of these investigations include:

1) The use of relatively expensive off-the-shelf, wired instrumentation and data processing hardware not designed specifically for SHM.
2) Excitation has, in general, been from ambient sources inherent to the operating environment and these sources tend to only excite the lower frequency global modes that are insensitive to local damage.
3) The data reduction is usually based on classical linear modal analysis and most studies assume that the structure can be modeled as a linear system before and after damage.
4) Statistical methods have not been used to quantify when changes in the dynamic response are significant and caused by damage. Varying environmental and operational conditions produce changes in the system’s dynamic response that can be easily mistaken for damage.

Taken as a whole, the aforementioned characteristics place serious limitations on the practical use of existing methodologies. Indeed, with the exception of applications to rotating machinery (Mitchell, 1992), there are, to the authors' knowledge, no examples of reliable strategies for SHM that are robust enough to be of practical use.

The goal of this research effort is to identify an appropriate application for SHM technology and to then develop a robust and cost-effective SHM system specific to this application by integrating and extending technologies from various engineering and information technology disciplines. Monitoring welded moment resisting steel frame connections in structures subjected to seismic excitation is the specific application that will be investigated. The SHM system will be composed of both hardware and software components. Changes in dynamic response resulting from damage will be detected with sensitive, dynamic response measurements made with *active* Micro-ElectroMechanical Systems (MEMS). Here the term *active* indicates that the sensing units will be designed to provide a local mechanical excitation source tailored to the monitoring activity. Software for data interrogation will incorporate statistical pattern recognition algorithms to identify that damage is present and data correlation techniques to locate the damage. The software will be integrated into the sensing unit through a programmable micro-processing chip. The processed data output of these sensing units will be monitored at a central location using a wireless data transmission system.

In this paper, the SHM problem is cast in the context of a statistical pattern recognition paradigm. In this paradigm, the SHM process can be broken down into four parts: (1) Operational Evaluation, (2) Data Acquisition and Cleansing, (3) Feature Extraction and Data Compression, and (4) Statistical Model Development for Feature Discrimination. Each steps of the statistical pattern recognition paradigm are addressed in the following sections.

OPERATIONAL EVALUATION

Before start implementing a monitoring system, it is essential to answer four questions regarding the implementation of a SHM system: (1) What are the life-safety and/or economic benefits provided by the monitoring?; (2) How is damage defined for the system being monitored?; (3) What are the operational and environmental conditions under which the system of interest functions?; and (4) What are the limitations on acquiring data in the operational environment? Operational evaluation begins to set the limitations on the monitoring as well as tailoring the monitoring to unique aspects of the system and unique damage features to be detected.

This study begins by identifying an application where structural health monitoring technology could provide a significant life safety and/or economic advantage over current inspection techniques. The application selected is the monitoring of welded connections in moment resisting steel frame structures that are subjected to seismic loading. This application is motivated by the costs, approximately $10,000, to remove and replace the architectural cladding and fire retardant to visually inspect a single joint after the Northridge and Kobe earthquakes (Darwin, 2000). A goal of this study is to develop a monitoring system that when mass produced can replace the visual inspection at a cost of $100 per joint while maintaining the same accuracy for detecting damaged connections. There are further economic benefits to be gained by accurate monitoring of these joints through timely reoccupation of the facility and the mitigation of additional economic losses associated with loss of revenues from extended facility evacuation. Finally, the SHM system can have a life safety benefit by providing information regarding the state of the structure after the main shock that, in turn, can be coupled with predictive models to evaluate the structure's ability to withstand aftershocks.

In this application damage will be defined as a crack through the thickness of the weld or girder The intent is to define a damage level that can be detected before it reaches a critical state that can lead to an abrupt failure of the structural system. A portion of this study will be aimed a quantifying the level of damage that can be accurately detected and determining if this level of damage is with the limits necessary to prevent system level failure of the structure.

The SHM system will be required to operate remotely in the presence of ambient vibrations and varying environmental conditions that the structure will see in it daily service. These conditions include varying live loads carried by the structure, varying boundary conditions that results from changing soil conditions and changing vibration environments caused by ground motion (aftershocks as well as from man-made sources), wind loading and mechanical equipment operating in the structure. Also, the system will have to be designed to operate without AC power, as there is typically a loss of power associated with seismic events. The primary limitation on acquiring data in the operating environment is to make the system small enough that it can be placed directly on the connection and behind the architectural cladding.

DATA ACQUISITION AND CLEANSING

The data acquisition portion of the SHM process involves selecting the number, types and location of sensors to be used along with the data acquisition hardware. Other considerations that must be addressed include how often the data should be collected and how to normalize the data.

To address the Deficiencies 1 and 2 listed in the Introduction, a major portion of this project will focus on developing an active sensing system specifically designed for SHM of welded moment resisting connections. There are several key components of this system; (1) micro-sensors with built-in a-to-d conversion capability, (2) wireless data transmission, (3) local data processing capabilities, (4) power source, and (5) a local excitation source. Fig. 1 shows the conceptual design for the proposed system. The components of the system are discussed below.

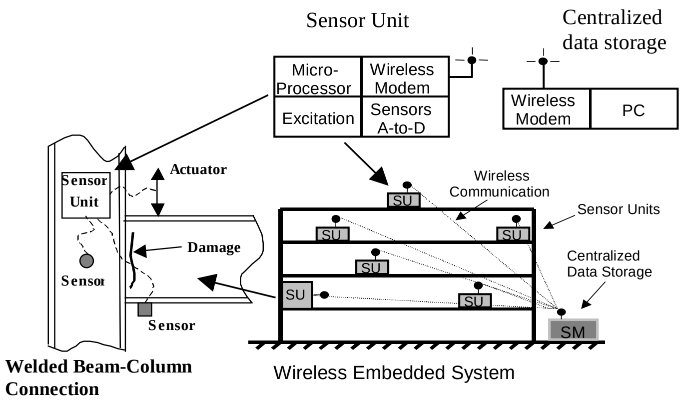

Figure 1 Research Objective: Move from a conventional wired sensing system to an active wireless system

Micro-sensors: MEMS accelerometers

Dynamic response will be sensed with MEMS accelerometers. MEMS sensors offer the advantage over conventional sensors of an approx. factor of 10-50 lower cost, very small size, relatively low power consumption and flexibility in design such that they can be tailored to the specific SHM activity. The low cost, resulting from fabrication methods similar to those used in semi-conductor manufacturing, will allow the sensor density to be significantly increased such that each joint can be directly monitored with a local sensing system consisting of several MEMS sensors.

Local excitation

The use of local excitation for SHM is a unique aspect of this research and will allow for a reliable SHM system that is as immune to unmeasured inputs as possible. Local excitation devices can overcome the problems of using ambient excitation sources that are not optimal for damage detection, and can aid in reducing the need for further data normalization. MEMS micro-machine actuators will be considered. These micromachines can provide an inexpensive, non-intrusive and repeatable local excitation source specifically designed to enhance the damage detection process. Excitations with MEMS devices, however are limited to small amplitude signals on the order of 10 N. Other sources of actuation including conventional hydro/pneumatic as well as magnetorestrictive actuators will also be considered as these actuators can provide forces of much higher magnitudes. Sandia National Laboratories has developed gas-transfer actuators in applications ranging from robotics to detonators. These actuators are essentially powered by a combustion process initiated from an electrical or optical source and can potentially provide forces greater than those from hydro/pneumatic or magnetorestrictive actuators. It should be noted that any of the aforementioned local excitation sources may not have the ability to provide forces large enough to open and close cracks in a structural joints.

Research issues that must be explored include the optimal waveform, the required excitation amplitude, and the power consumption of the actuator. Candidate excitation methods utilizing various signals such as swept sine input or chirp signals will be investigated. A homodyne detection scheme is one way by which local actuation can be physically integrated into SHM (Farrar, et al., 1999). The detection scheme is essentially an amplitude modulation/demodulation process much like the one employed for radio transmission. A predetermined carrier signal is sent by the actuator and is received by a nearby sensor. The received signal is demodulated by convolving it with the same carrier wave provided by the actuator and then low-pass filtering the output of that convolution. The resulting signal is a measure of the structure’s modulating effects on the original carrier wave signal and is typically called the transmissibility. The most significant advantage of a homodyne detection scheme is the ability to tailor the carrier wave signal to the specific application and the ability to reject extraneous response components not directly related to the prescribed input. The optimal carrier wave will be one that is most sensitive to damage-induced changes in the system.

Wireless data transmission

Initial attempts at deploying SHM systems in the field have shown that the single biggest maintenance issue is associated with wiring the sensors to a central data acquisition system. Studies of SHM systems deployed in the field have shown that the annual maintenance costs associated with wiring are approximately 10% of the system cost (Nigbor and Diehl, 1997). This project will make use of existing very compact, low-power, RF telecommunications hardware being commercialized by Crossbow, Inc (www.xbow.com). These systems have the necessary range, bandwidth, and power for the SHM application being considered in this study.

Local Data Processing

With a monitoring system based on wireless communication and embedded systems, it is possible to perform a portion of the data processing locally with an embedded microprocessor. Local data processing can alleviate issues with signal time synchronization when the sensor array is spread out over large distances. Local processing can also enhance the ability to perform near real time structural integrity assessment as most of the data processing can be done in parallel by the local microprocessors. This local processing minimizes the amount of information that must be transmitted to the central monitoring station and produces a more fault-tolerant monitoring system. Also, in terms of power consumption, it is much more expensive to transmit data than it is to process the data. Therefore, the local data processing and subsequent transmission of compressed information minimize the power demands of the sensing system. The concept of pushing data analysis forward is fundamental to this integrated SHM system and represents a departure from the conventional instrumentation design and computational strategies for SHM.

Power Source

All sensing/actuation schemes will require a power supply. For the steel frame structures application AC power may not be available after a seismic event. Research is underway to use the energy from ambient vibrations source harvested through PZT films that are mounted to the surface of beams in the structure (Cattan, 1999). The building application is particularly well suited for this because of the abundant surfaces available for mounting such materials. In addition, methods for amplifying the motion through the use of secondary structures mounted on the main girders will be developed. Furthermore, new sensors and actuators are being designed to operate only when triggered by certain signals, thereby reducing their power consumption. This research will consider the use of materials that extract power from ambient vibration sources, methods to store that power, and the controlled release of this power as needed. A key issue that must be addressed is powering the local excitation source.

Data Normalization

All real-world structures will be subjected to changing environmental and operational states such as varying temperature, moisture, and dead- and live-loading conditions. In the context of SHM, data normalization refers to process of minimizing the influence of changing operational and environmental conditions on the damage detection process. These changing conditions will affect the measured data features that are used to define the undamaged or "normal condition" of the structure. In this case, there may be a continuous range of normal conditions, and it is clearly undesirable for the SHM system to signal damage simply because of a change in the environment or operation. In fact, these changes can often mask subtler structural changes caused by damage (Sohn et al., 2001).

One approach to solving this problem is to measure parameters related to these environmental and operational conditions, as well as the data features, over a wild range of these varying conditions to characterize the normal conditions. The normal conditions can be then parameterized to reflect the different environmental and operational states. A damage detector, which does not provide false indication of damage under changing environmental and operational conditions, is then built. On the other hand, there are cases where it is difficult to measure parameters related to the environmental and operational conditions.

The approach taken in this study to minimize the influence of environmental and operational variability has two components. First, the local actuation scheme attempts to provide a tailored input that exceeds the influence of the other extraneous inputs. Second, a data interrogation scheme is used to identify the underlying functional relationship between the data features and the influential, but unmeasured, environmental and operational parameters. This idea is based on auto-associative neural networks where target outputs of the data interrogation process are also the inputs to the network. Using the features derived from measured data corresponding to the normal conditions, the auto-associative neural network is trained to characterize the underlying dependency of the these features on the unmeasured environmental and operational variations. This dependency is identified by treating these environmental and operational conditions as hidden intrinsic variables in the neural network. The result of the process is a functional relationship between the features and the assumed influential environmental and operational conditions. A challenge in developing the auto-associative neural network mapping is to properly assume the number of influential environmental and operational parameters.

FEATURE EXTRACTION AND DATA COMPRESSION

Feature extraction is the process of the identifying damage-sensitive properties from the measured vibration response. Almost all feature extraction procedures inherently perform some form of data compression. Data compression into feature vectors of small dimension is necessary if accurate estimates of the features' statistical distributions are to be obtained (Bishop, 1995).

The goal in this portion of the SHM system development is to take advantage of the fact that for most real world structures the damage scenarios cause the structure to exhibit nonlinear and/or non-stationary response. As an example, a time-frequency response plot measured on a cantilever beam in an undamaged and damaged state, Fig. 2, shows harmonic generation and non-stationary response, both of which are indicative of a crack opening and closing. A novel time series analysis has been developed to locate damage sources in a mechanical system, which is running in various operational environments. The source of damage is located by solely analyzing the acceleration time histories recorded from a structure of interest. First, an additional data normalization procedure is used. This procedure selects a reference signal, which is "closest" to a newly obtained signal, from an ensemble of signals recorded when the structure is undamaged. Second, a two-stage prediction model, combining Auto-Regressive (AR) and Auto-Regressive with eXogenous inputs (ARX) techniques, is constructed from the selected reference signal. Then, the residual error, which is the difference between the actual acceleration measurement for the new signal and the prediction obtained from the AR-ARX model developed from the reference signal, is defined as the damage-sensitive feature. This approach is based on the premise that if there were damage in the structure, the prediction model previously identified using the undamaged time history would not be able to reproduce the newly obtained time series measured from the damaged structure. Furthermore, the increase in residual errors would be maximized at the sensors instrumented near the actual damage locations. The applicability of this approach has been demonstrated using acceleration time histories obtained from an eight degrees-of-freedom (DOFs) mass-spring system (Sohn and Farrar, 2001).

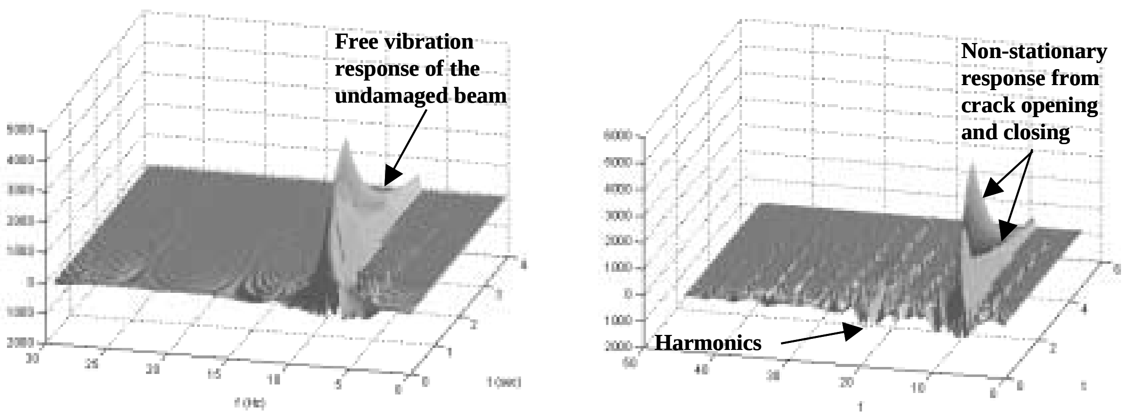

Figure 2: Wigner-Ville time-frequency transforms of the free-vibration acceleration-time histories measured on an uncracked cantilever beam (left) and a cracked cantilever beam (right).

STATISTICAL MODELING FOR FEATURE DISCRIMINATION

Statistical modeling for feature discrimination is the implementation of algorithms that analyze distributions in the extracted features in an effort to determine the damage state of the structure. Statistical modeling is used to quantify when changes in the data features can be considered significant. Also, these algorithms are used to discriminate between changes in the features caused by damage and changes caused by varying operational and environmental conditions.

The algorithms used in statistical modeling fall into the three general categories: (1) group classification, (2) regression analysis, and (3) outlier detection. The appropriate algorithm to use will depend on the ability to perform *supervised* or *unsupervised* learning. Here, supervised learning refers to the case were examples of data from damaged and undamaged structures are available. Unsupervised learning refers to the case were data are only available from the undamaged structure. Economic and life-safety concerns typically make it difficult to obtain data from damaged aerospace, civil or mechanical infrastructure and, hence, necessitate an unsupervised learning approach. The development of unsupervised learning techniques based statistical process control (Montgomery, 1996), principal component analysis (Ripley, 1996), and linear and quadratic discriminants (Fukunaga, 1990) will be the areas of study for this portion of the research effort.

Damage identification is a problem that can be addressed at many levels. Stated in its most basic form, the objective is to ascertain simply if damage is present or not. One class of unsupervised learning algorithms, which show considerable promise for this purpose, is grouped under the name *outlier detection methods* (Barnett and Lewis, 1994). The philosophy is simple; during the normal operation of a system or structure, measurements are recorded and features are extracted from data, which characterize the normal conditions. After training the diagnostic procedure in question, subsequent data can be examined to see if the features deviate significantly from the norm. That is, outlier detection is a technique for deciding if measurements from a system or structure indicate departure form previously established normal conditions. An alarm is signaled if the index value increased above a pre-determined threshold.

The case of outlier detection in univariate data is relatively straightforward in that outliers must "stick out" from one end or the other of the data set distribution. There are numerous discordance tests but one of the most common, and the one whose extension to multivariate data will be employed later, is based on deviation statistics and given by

$$z_{\varsigma} = \frac{x_{\xi} - \overline{x}}{s} \tag{1}$$

where x_ς is the potential outlier, and $\bar{x}$ and s are the sample mean and standard deviation, respectively. The latter two values may be calculated with or without the potential outlier in the sample depending upon whether *inclusive* or *exclusive* measures are preferred. This discordance value is then compared to some threshold value to identify an outlier.

In general, a multivariate data set consisting of *n* observations in *p* variables may be represented as *n* points in *p*-dimensional object space. It becomes clear that detection of outliers in multivariate data is more difficult than in the univariate case due to the potential outlier having more "room to hide". The discordance test, which is the multivariate equivalent of Equation 1, is the Mahalanobis squared distance measure given by,

$$D_\varsigma = (\mathbf{x}_\xi - \overline{\mathbf{x}})^{\mathrm{T}} \mathbf{s}^{-1} (\mathbf{x}_\xi - \overline{\mathbf{x}}) \tag{2}$$

where $\mathbf{x}_\varsigma$ is the potential outlier vector, $\overline{\mathbf{x}}$ is the sample mean vector of and $\mathbf{s}$ the sample covariance matrix.

As with the univariate discordance test, the mean and covariance may be inclusive or exclusive measures. In many practical situations the outlier is not known beforehand and so the test would necessarily be conducted inclusively. In the case of on-line damage detection the potential outlier is, however, always known beforehand. (It is simply the most recent sampled point). Therefore, it is more sensible to calculate a value for the Mahalanobis squared distance without this observation contaminating the statistics of the normal data. Whichever method is used, the Mahalanobis squared distance of the potential outlier is checked against a threshold value and the status of the structure is determined based on this comparison.

Determination of the rejection threshold is critical. This value is dependent on both the number of observations and the dimension of feature space being selected. A Monte Carlo method was used here to arrive at the threshold value. The procedure for this method was to construct a $p \times n$ (dimension of feature space $\times$ number of observations) matrix with each element being a randomly generated number from a normal distribution with zero mean and a unit variance. The Mahalanobis squared distances were calculated for all the *p*-vector components, using Equation 2 where $\overline{\mathbf{x}}$ and $\mathbf{s}$ are inclusive measures, and the largest value stored. This process was repeated for at least 1000 trials whereupon the array containing all the largest Mahalanobis squared distances was then ordered in terms of magnitude. The critical values for the 5% and 1% tests of discordance are given by the Mahalanobis squared distances in the array above which 5% and 1% of the trials occur.

Note that there is an implicit assumption throughout that the normal condition set has a Gaussian distribution. This assumption will not generally be true. However, if the deviations from the normality are small, i.e. the true distribution is uni-modal and has appropriately weighted tails, the outlier analysis may work very well. If the normal condition set is multi-modal or deviates significantly from a Gaussian distribution, other methods based on density estimation or neural networks would be generally used.

SYSTEM VERIFICATION

As sensing hardware and feature extraction and statistical discrimination algorithms are developed, they will be tested on a variety of systems. These systems will include "bench-top" physical systems that can capture the salient features of a damaged structural connection. Computer simulations with the finite element method provide a cost-effective method to generated numerical data to which the data interrogation algorithms can be applied. However, the end goal of this program is to demonstrate a prototype system on a "real world" steel frame system within two years. For this goal it will, most likely, be necessary to seed damage in the structure to test the ability of the system to identify damage.

SUMMARY

This paper outlines the development of a monitoring system that integrates hardware and software components to monitor welded connections in steel moment-resisting frame structures. This application is selected based on the known susceptibility of these joints to cracking during seismic loading and the high cost of visual inspection of these joints. Hardware components of this system are currently being developed jointly with faculty from Stanford University's Mechanical and Civil Engineering Departments. Software for data interrogation and statistical pattern recognition techniques is currently developed at Los Alamos National Laboratory, and has been applied to various data sets with considerable success. Although this integrated system is being developed for a specific application, any damage diagnosis problem can be cast in the context of the statistical pattern recognition paradigm presented in this paper. Therefore, it is the authors' hope that this paper can be used as a guideline for developing a structural health monitoring system. Finally, the most challenging issues foreseen in the project are the development of a local excitation and a powering system for the actuators and sensing devices. This self-powering capability will be critical for cases where AC power will not be available after an extreme event such as earthquake.

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